

## Seminario de Análisis Numérico y Modelamiento Matemático de Estudiantes



### *Conservative numerical scheme for dissipative wave equation with fractional derivative damping*

In this work, we present a numerical approach for studying the asymptotic behavior of a general wave equation with a dissipative fractional derivative term. The Caputo fractional derivative and its exponential generalization, used as models for the dissipative effect in evolution equations, have several applications, such as in thermal stresses, models of porous electrodes, relaxation vibrations, and viscoelasticity. This nonclassical derivative appears as a nonlocal term and is generally approximated using quadrature formulas, some of which can be quite accurate due to their high order, but do not produce a constant decay of energy, as occurs in the continuous case.

To address this issue and eliminate oscillations, we introduce an augmented Mbodje model[1], which we discretize while preserving energy decay. We apply our results to several vibration and elasticity models. [2][3][4][5]

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**Dr. Mauricio Sepúlveda Cortés**  
Universidad de Concepción

Departamento de Ingeniería Matemática, UdeC  
Centro de Investigación en Ingeniería Matemática (CI<sup>2</sup>MA), UdeC  
Centro de Modelamiento Matemático de la Universidad de Chile

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Auditorio Hermann Alder  
Weller, CI<sup>2</sup>MA, UdeC